ENGINEERING ECONOMICS ISE460
SESSION 3
CHAPTER 3, May 30, 2011

OUTLINE

- **QUESTIONS?**
- **News?**
- **CHAPTER 3 – Time is Money – That's it in a nutshell!**

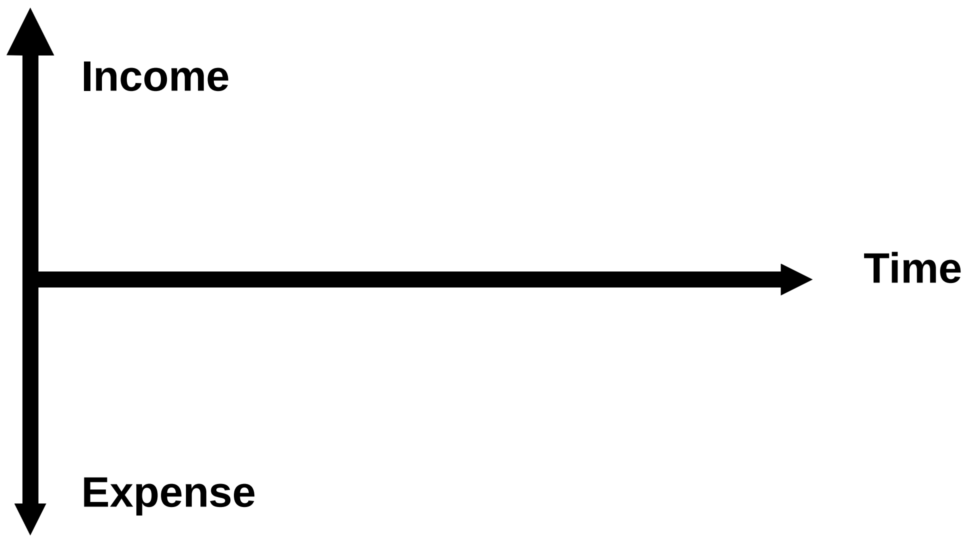
Overview

- **EXCEL DEMO OF CHAPTER 3 MATERIAL**
- **CHAPTER 3 ESSENTIALS**
  - **FORMULAS**
  - **EXAMPLES 3.25, 3.26, 3.27**
  - **TABLES**

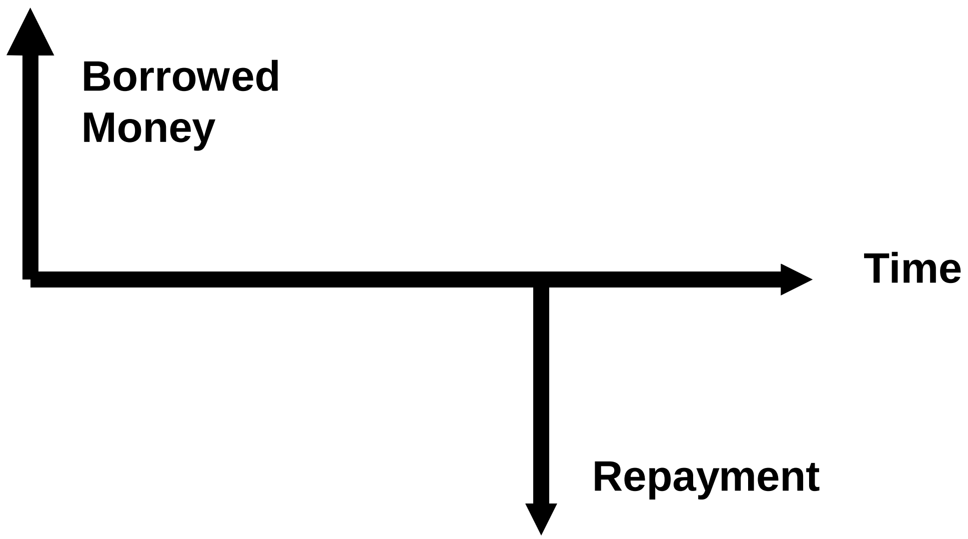
The time value of money

- If you have money, you can either
  - Lend it and get interest
  - Invest it and hope you get interest and that you get it back
  - Invest it and hope you can resell it for more
  - Use it to buy something or a service
- In the first two instances, when you ask for it back in the future, you'll get your money plus what it earned. In other words, it will have grown
- Unfortunately, during the same time period, money in general will have lost some of its value due to inflation
- But if you invest wisely, the net should be higher
- In this chapter we assume that the two effects are combined in the quoted rates

Graphical Representation

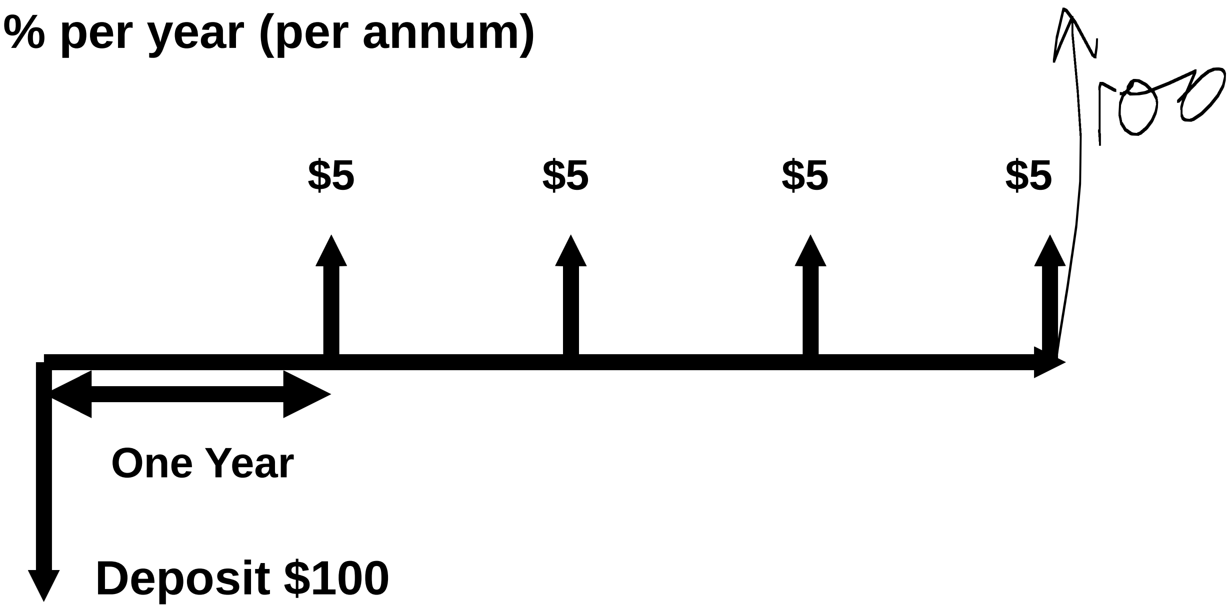


Graphical Representation - Example



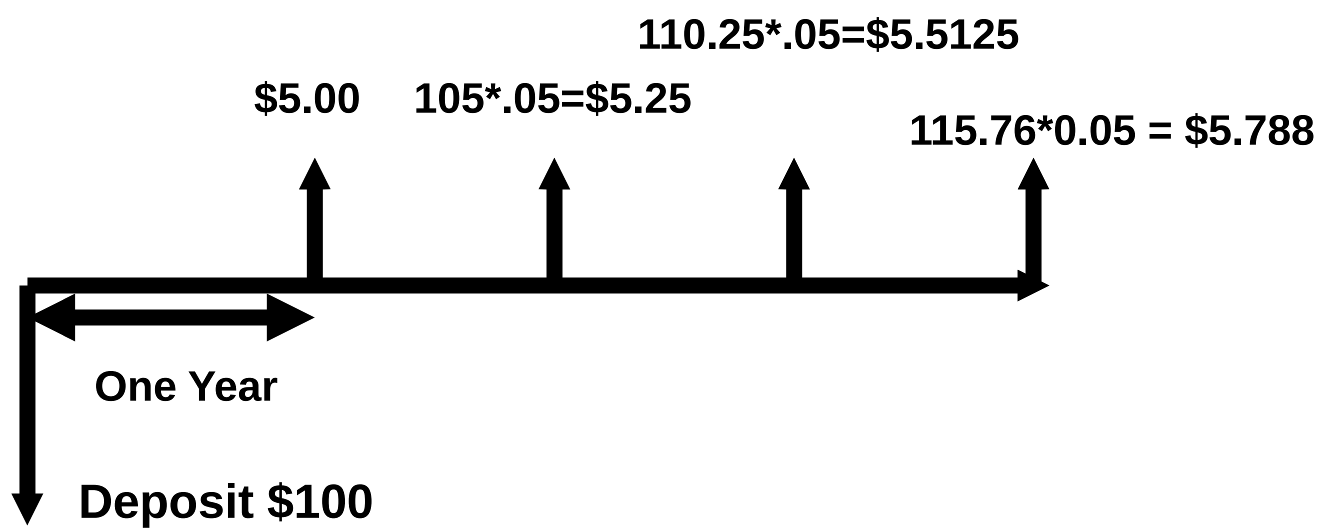
Simple Interest

- **At 5 % per year (per annum)**



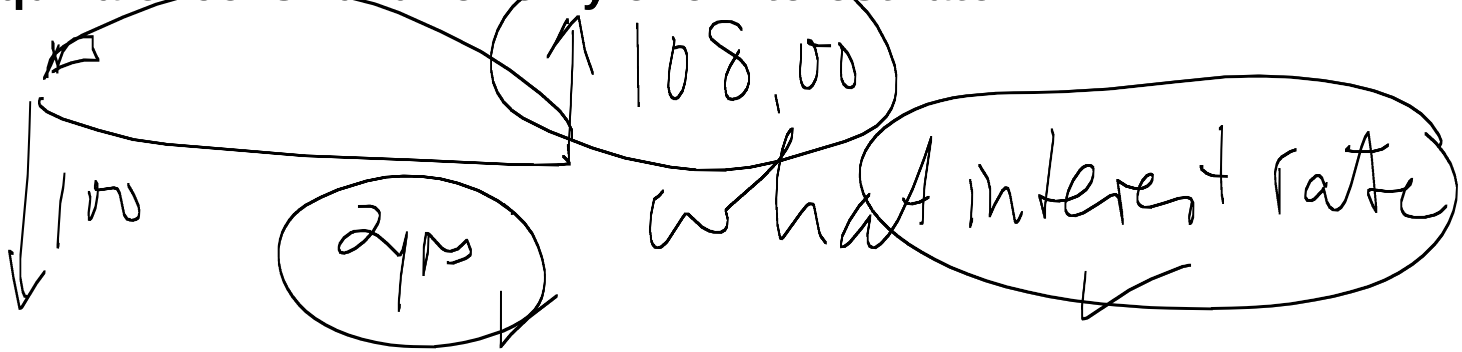
Compound Interest

- At 5 % per year (per annum), compounded annually



Equivalence

- **We can compare different cash flows by comparing their value at the same time**
- **The choice of time is arbitrary, but is usually the present or some date in the future**
- **Equivalence is valid for only one interest rate**



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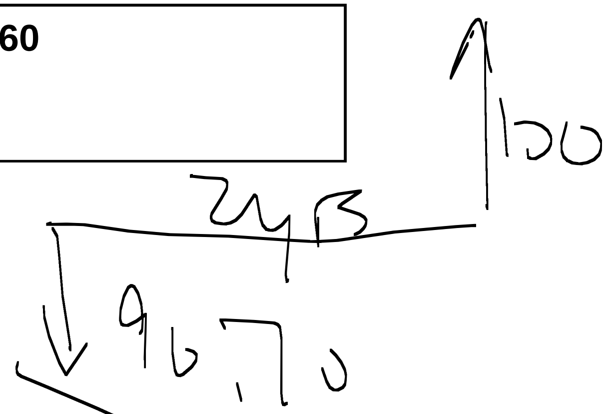
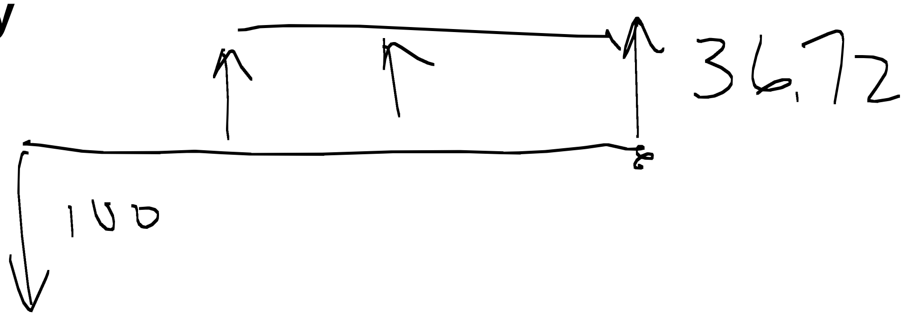
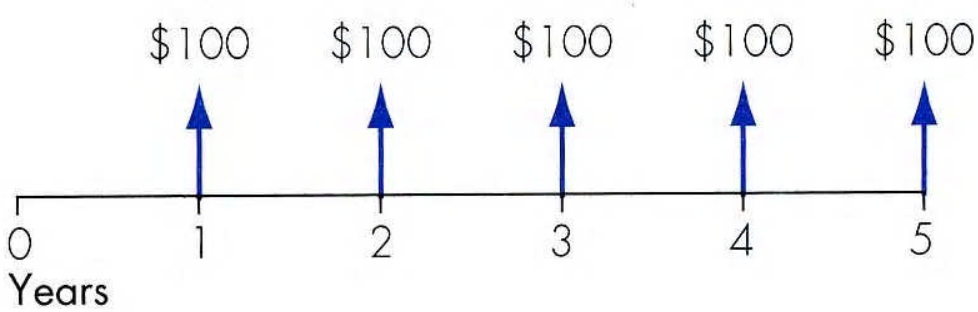


Equivalence - example

- **@ 5% compound interest**
- **$100 held as cash for 3 years will be worth $86.38**
- **$100 loaned today can be paid back in 3 equal annual installments of $36.72**
- **$100 promised to be paid 2 years from now is worth $90.70 today**

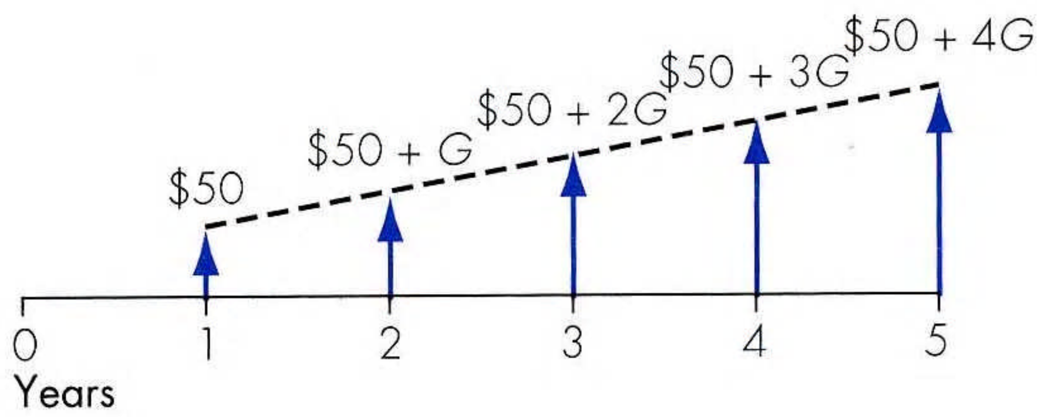


Uniform Series



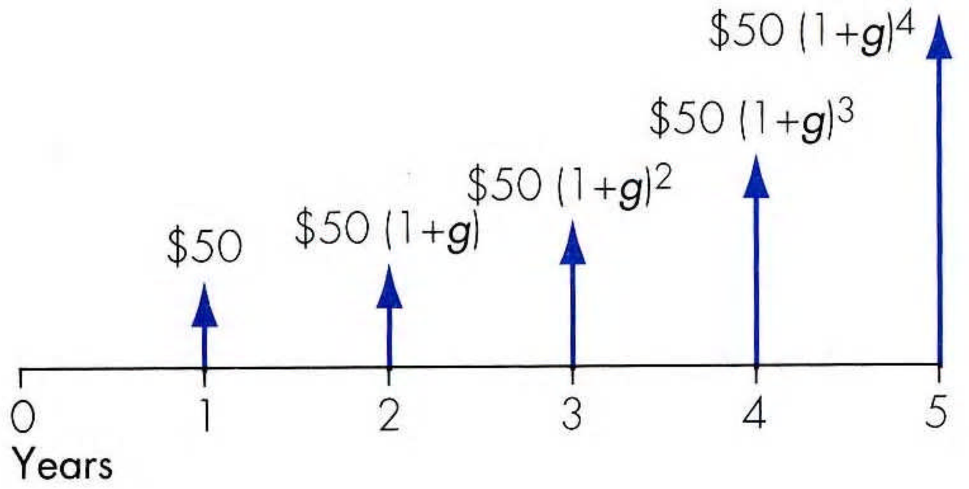
(b) Equal (uniform) payment series at regular intervals

Linear Gradient



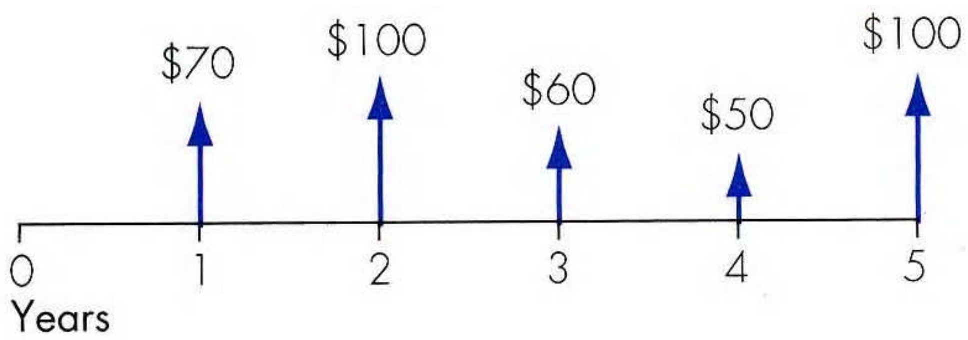
(c) Linear gradient series, where each cash flow in a series increases or decreases by a fixed amount, *G*

Geometric Gradient



(d) Geometric gradient series, where each cash flow in a series increases or decreases by a fixed rate (percentage), g

Irregular Series



(e) Irregular payment series, which exhibits no regular overall pattern

Notation

- A_n = payment or receipt occurring at the end of period n
- A = a constant payment or receipt at end of each period
- N = number of periods
- i = interest rate per interest period
- P = a sum of money at time zero = Present value or worth
- F = a sum of money at some future date (usually the end of the time span that you are analyzing

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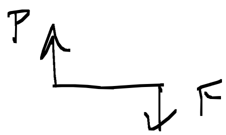
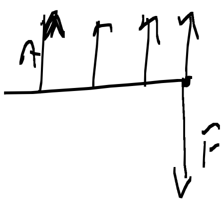
CHAPTER 3 FORMULAS

Name	Formula	Excel Function
Single Payment		
Compound Amount Factor (compounding)	$F = P(1+i)^N$	FV(i, N, 0, P)
Present Worth Factor (discounting)	$P = F(1+i)^{-N}$	PV(i, N, 0, F)
Equal Payments		
Compound Amount Factor	$F = A\left[\frac{(1+i)^N - 1}{i}\right]$	FV(i, N, A)
Sinking Fund Factor	$A = F\left[\frac{(1+i)^N - 1}{i}\right]^{-1}$	PMT(i, N, 0, F)
Capital Recovery Factor (Annuity Factor)	$A = P\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]^{-1}$	PMT(i, N, P)
Present Worth Factor	$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$	PV(i, N, A)





CHAPTER 3 FORMULAS (CONTINUED)

Linear Gradient Series

Present Worth Factor	$P = G\left[\dfrac{i(1+i)^N - iN - 1}{i^2(1+i)^N}\right]$	none
Gradient to equal payment conversion Factor	$A = G\left[\dfrac{(1+i)^N - iN - 1}{i[(1+i)^N - 1]}\right]$	none

Geometric Gradient Series

Present Worth Factor	$P = A_1\left[\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}\right], \quad i \neq g$ $P = NA_1/(1+i), \quad i = g$	none
Future Worth Factor	$F = A_1\left[\dfrac{(1+i)^N - (1+g)^N}{i-g}\right], \quad i \neq g$ $F = NA_1/(1+i)^{N-1}, \quad i = g$	none

Derivations

- **DERIVATION OF SOME FORMULAS**
  - **SINGLE CASH FLOW**
  - **UNIFORM SERIES OF PAYMENTS**
  - **LINEAR GRADIENT SERIES**
  - **GEOMETRIC GRADIENT SERIES**
  - **IRREGULAR SERIES**
- **GO OVER REMAINING CHAPTER 3 EXAMPLES**

Formulas

F – future value

P – present value

N – number of periods

i – interest rate per period

A – payment at the end of each period

G – increase in payment per period starting in the second

A_1 – Initial payment

n – Index for periods

g –percentage change from payment to payment

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Formulas – Future value, Present value

1) $F = P(1+i)^N$

2) $\dfrac{F}{P} = (1+i)^N$

F/P find F, given P

3) $\dfrac{P}{F} = (1+i)^{-N}$

4) $P = F(1+i)^{-N}$

5) $\log\dfrac{\mathrm{P}}{\mathrm{F}} = -N\log(1+i)$

6) $N = \dfrac{-\log\left(\dfrac{P}{F}\right)}{\log(1+i)}$

Formulas -- Payments

$$7)\quad F = A + A(1+i) + A(1+i)^2 + \ldots + A(1+i)^{N-1}$$

$$8)\quad (1+\mathrm{i})F = A(1+i) + A(1+i)^2 + \ldots + A(1+i)^N$$

$$8) - 7)\quad F(1+i) - F = -A + A(1+i)^N$$

$$9)\quad iF = A\left[(1+i)^N - 1\right]$$

$$10)\quad F = A\left[\frac{(1+i)^N - 1}{i}\right]$$

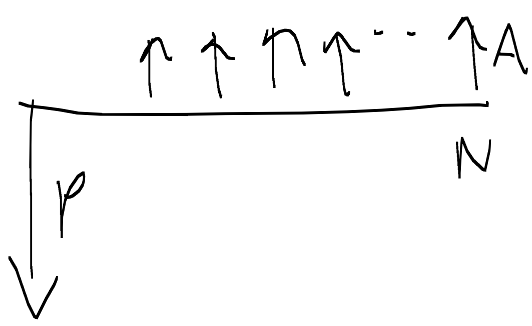
$$11)\quad A = F\left[\frac{i}{(1+i)^N - 1}\right]$$

Formulas – Payments (continued)

$$12)\text{ into }4)\quad P = A\left[\frac{(1+i)^N - 1}{i}\right](1+i)^{-N}$$

$$13)\quad P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$$14)\quad A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$



Formulas – Gradient Series

Gradient Series

$$15)\quad P_{TOT} = A_1(1+i)^{-1} + A_1(1+i)^{-2} + \dots + A_1(1+i)^{N-1} +$$
$$0 + G(1+i)^{-2} + 2G(1+i)^{-3} + \dots + (N-1)G(1+i)^{-N}$$

$$16)\quad P_G = \sum_{j=1}^{N} (j-1)G(1+i)^{-N}$$

$$17)\quad P_G = G\left[\frac{(1+i)N - iN - 1}{i^2(1+i)^N}\right]$$

17) into 14)

$$18)\quad A = G\left[\frac{(1+i)^N - iN - 1}{i\left[(1+i)^N - 1\right]}\right]$$

18) into 10)

$$19)\quad F = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i} - N\right]$$

Formulas -- Geometric Gradient Series

- **Geometric Gradient Series**

$$20)\quad A_n = A_1(1+g)^{n-1}$$

$$21)\quad P_n = A_n(1+i)^{-n}$$

20) into 21)

$$22)\quad P_n = A_1(1+g)^{n-1}(1+i)^{-n}$$

Summing over all payments:

$$23)\quad P = \sum_{n=1}^{N} A_1(1+g)^{n-1}(1+i)^{-n}$$

$$24)\quad P = \frac{A_1}{(1+g)}\sum_{n=1}^{N}\left[\frac{1+g}{1+i}\right]^n$$

Formulas -- Geometric Gradient (continued)

$$a = \frac{A_1}{(1+g)}$$

$$x = \frac{1+g}{1+i}$$

$$25)\quad P = a(x + x^2 + \ldots + x^N$$

multiplying by x :

$$26)\quad xP = a(x^2 + x^3 + \ldots + x^{N+1}$$

$$26) - 25)$$

$$P - xP = a(x - x^{N+1})$$

$$P = \frac{a(x - x^{N+1})}{1-x} \quad \text{Note x cannot} = 1$$

$$27)\quad P = \left\{ \begin{array}{ll} A_1\left[\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}\right], & i \neq g \\ NA_1/(1+i) & i = g \end{array} \right\}$$

Formulas -- Geometric Gradient (continued)

1) into 27)

$$28)\quad F = \begin{cases} A_1\left[\dfrac{-(1+g)^N+(1+i)^N}{i-g}\right], & i \neq g \\ NA_1(1+i)^{N-1} & i = g \end{cases}$$

Tables

870 APPENDIX A Interest Factors for Discrete Compounding

0.25%

	Single Payment		Equal Payment Series			
N	Compound Amount Factor (F/P,i,N)	Present Worth Factor (P/F,i,N)	Compound Amount Factor (F/A,i,N)	Sinking Fund Factor (A/F,i,N)	Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)
1	1.0025	0.9975	1.0000	1.0000	0.9975	1.0025
2	1.0050	0.9950	2.0025	0.4994	1.9925	0.5019
3	1.0075	0.9925	3.0075	0.3325	2.9851	0.3350
4	1.0100	0.9901	4.0150	0.2491	3.9751	0.2516
5	1.0126	0.9876	5.0251	0.1990	4.9627	0.2015
6	1.0151	0.9851	6.0376	0.1656	5.9478	0.1681
7	1.0176	0.9827	7.0527	0.1418	6.9305	0.1443
8	1.0202	0.9802	8.0704	0.1239	7.9107	0.1264
9	1.0227	0.9778	9.0905	0.1100	8.8885	0.1125
10	1.0253	0.9753	10.1133	0.0989	9.8639	0.1014

TABLE 3.2 Typical Repayment Plans for a Bank Loan of $20,000 (for $N = 5$ years and $i = 9\%$)

	Repayments		
	Plan 1	Plan 2	Plan 3
Year 1	$ 5,141.85	0	$ 1,800.00
Year 2	5,141.85	0	1,800.00
Year 3	5,141.85	0	1,800.00
Year 4	5,141.85	0	1,800.00
Year 5	5,141.85	$30,772.48	21,800.00
Total of payments	$25,709.25	$30,772.48	$29,000.00
Total interest paid	$ 5,709.25	$10,772.48	$ 9,000.00

Plan 1: Equal annual installments; Plan 2: End-of-loan-period repayment of principal and interest; Plan 3: Annual repayment of interest and end-of-loan repayment of principal

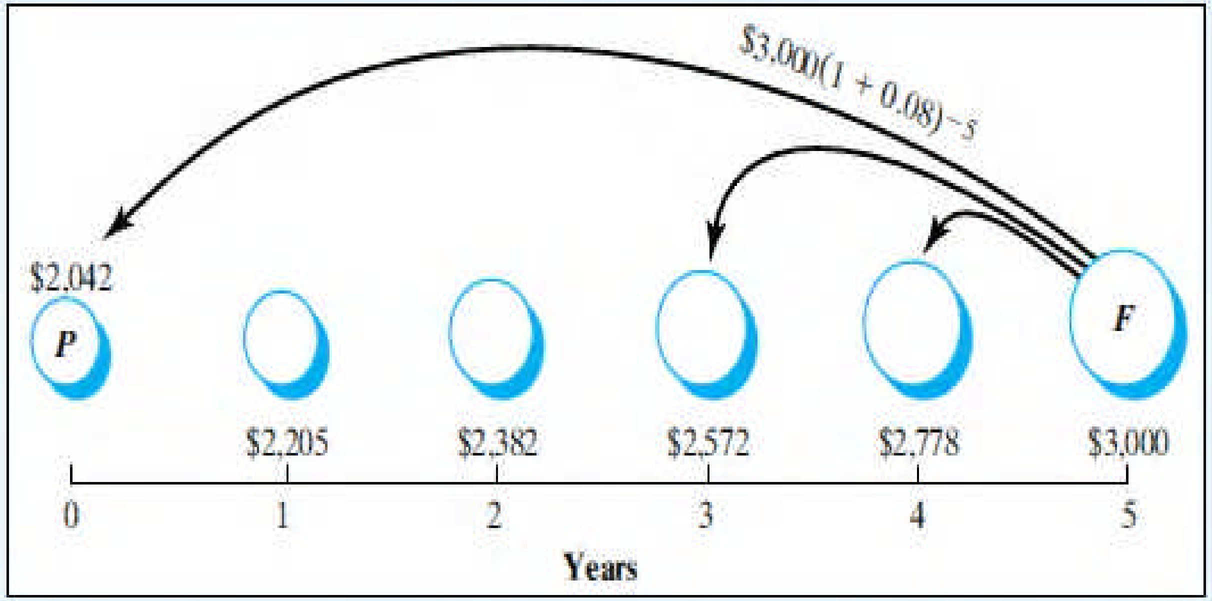
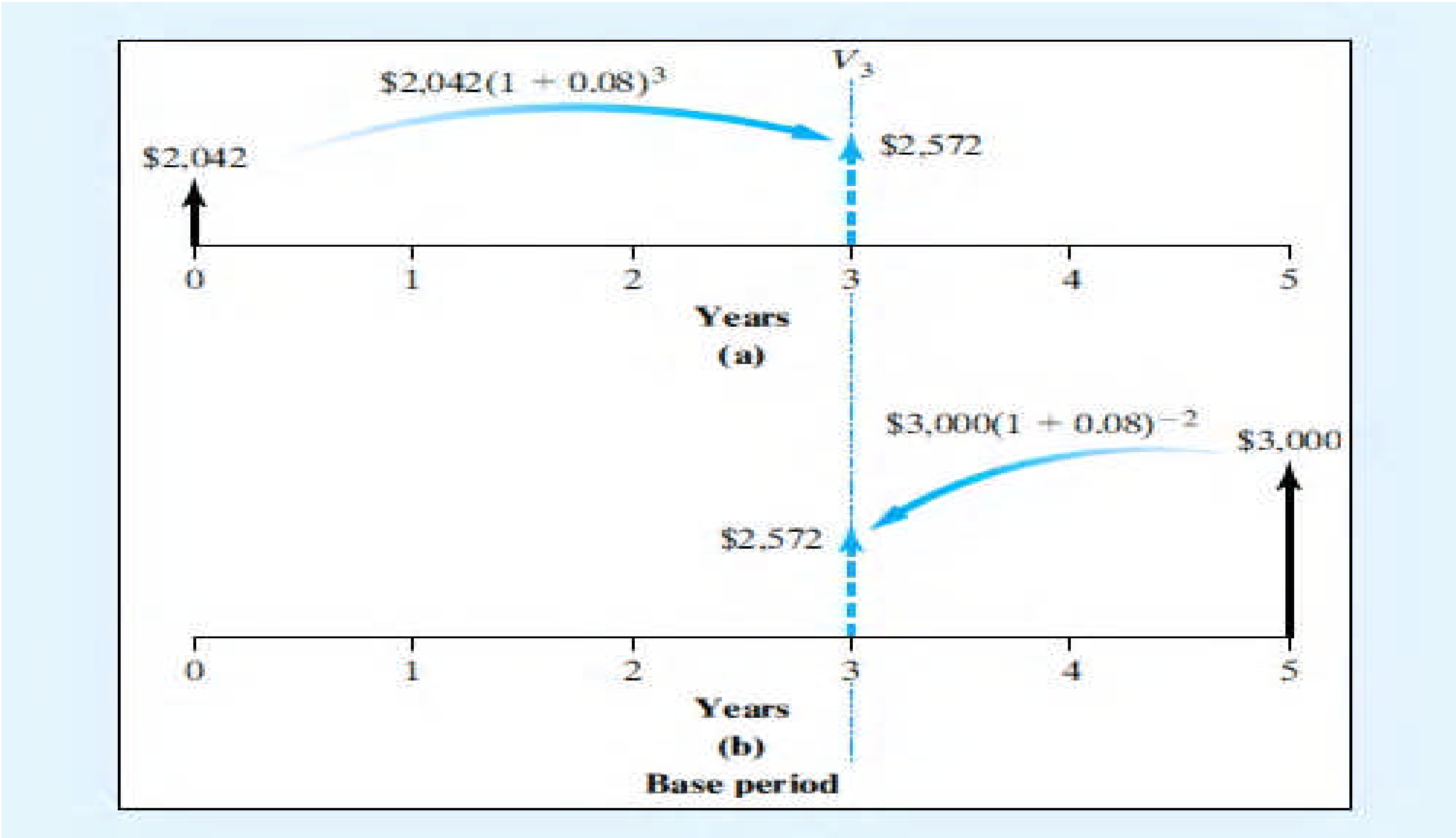


Figure 3.8 Various dollar amounts that will be economically equivalent to $3,000 in five years, given an interest rate of 8% (Example 3.3).

Equivalence at any point in time



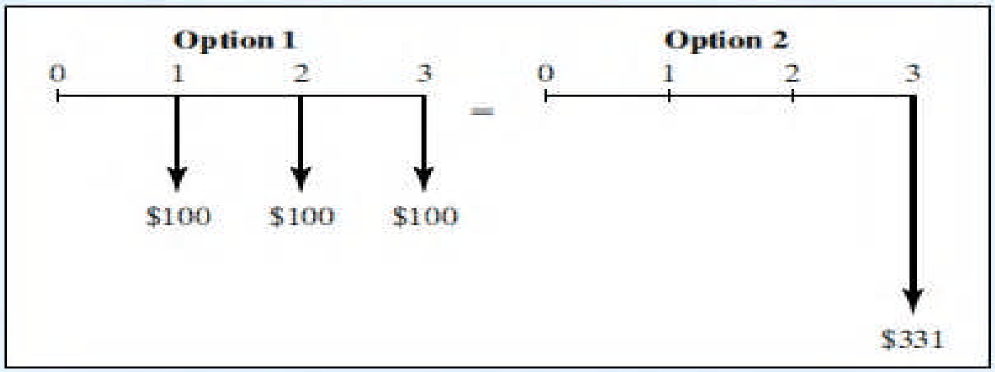
EXAMPLE 3.6 Equivalence Calculations with Multiple Payments

Suppose that you borrow $1,000 from a bank for three years at 10% annual interest. The bank offers two options: (1) repaying the interest charges for each year at the end of that year and repaying the principal at the end of year 3 or (2) repaying the loan all at once (including both interest and principal) at the end of year 3. The repayment schedules for the two options are as follows:

Options	Year 1	Year 2	Year 3
• Option 1: End-of-year repayment of interest, and principal repayment at end of loan	$100	$100	$1,100
• Option 2: One end-of-loan repayment of both principal and interest	0	0	1,331

Determine whether these options are equivalent, assuming that the appropriate interest rate for the comparison is 10%.

Example 3.6 - solution



$$
\begin{aligned}
F_3 \text{ for } \$100 \text{ at } n &= 1: \$100(1 + .10)^{3-1} = \$121;\\
F_3 \text{ for } \$100 \text{ at } n &= 2: \$100(1 + .10)^{3-2} = \$110;\\
F_3 \text{ for } \$100 \text{ at } n &= 3: \$100(1 + .10)^{3-3} = \underline{\$100};\\
\text{Total} &= \$331.
\end{aligned}
$$

By converting the cash flow in Option 1 to a single future payment at year 3, we can compare Options 1 and 2. We see that the two interest payments are equivalent. Thus, the bank would be economically indifferent to a choice between the two plans. Note that the final interest payment in Option 1 does not accrue any compound interest.