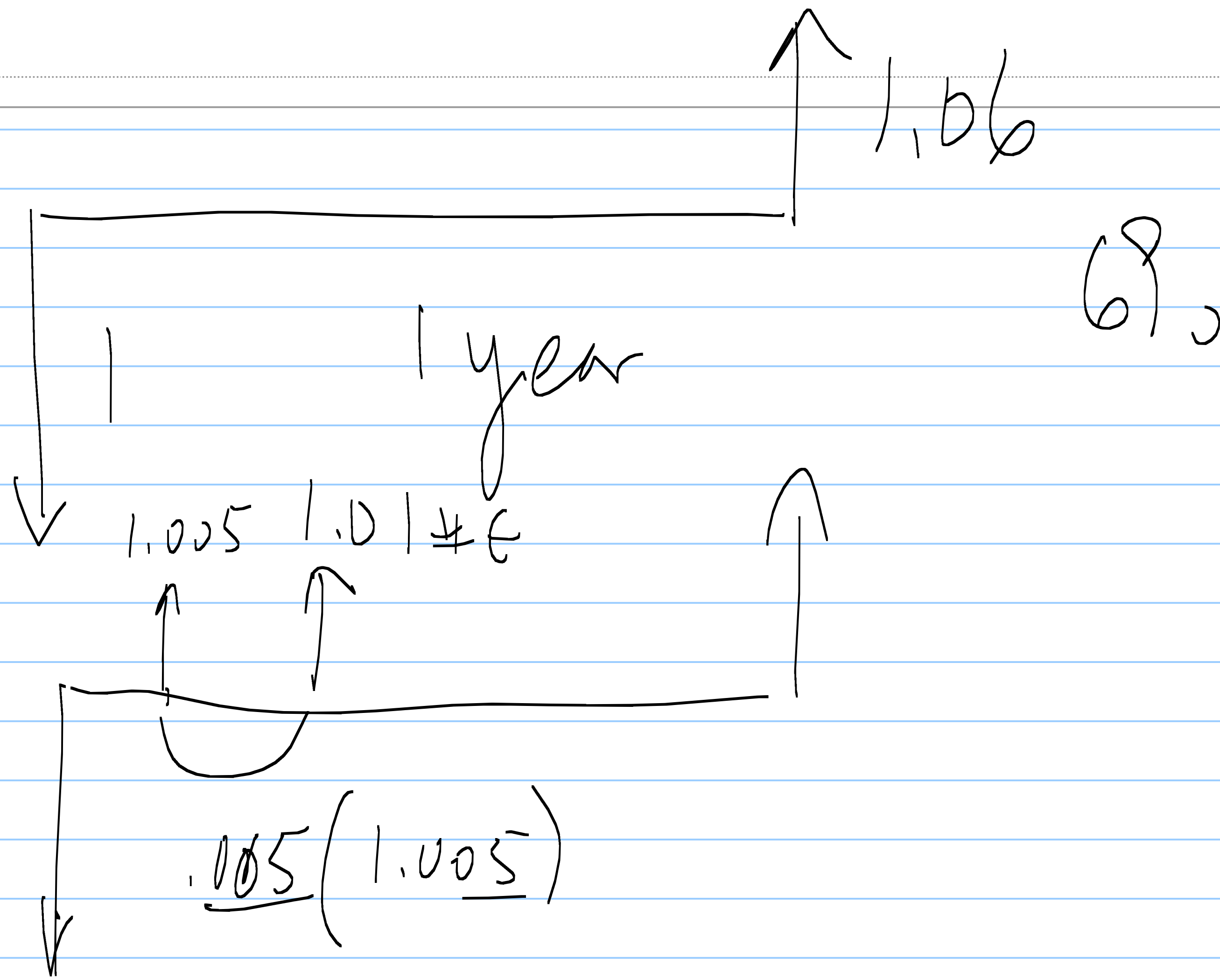
Note Title 5/31/2011



Note Title 5/31/2011

F

P

5 = n years

interest rate = 6%

$$F = P(1+i)^n$$

$$F = P(1+.06)^5$$

$$F = P(1.3382)$$

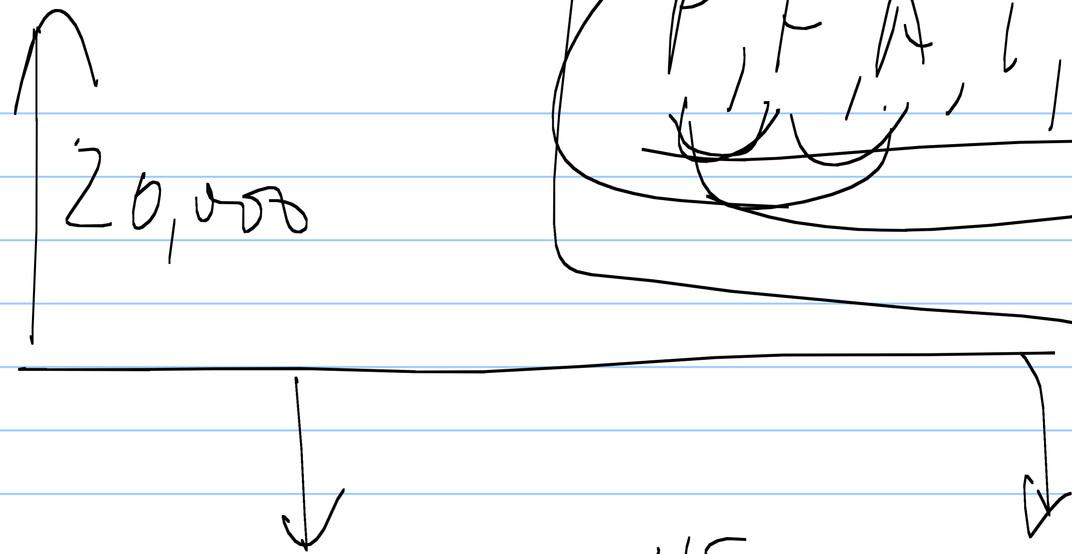
100

$$F = P\left(1+\frac{.06}{12}\right)^{5\times 12}$$

~~1.4~~

~~3~~

~~1.6~~



P, F, A, i, n

4 3 known

48

$$(.0235)\,20{,}000 = \$470.00$$

$$\frac{470.00 \times 48}{(\qquad)}$$

25,000
5,000

owe 20,000

at .5%/month

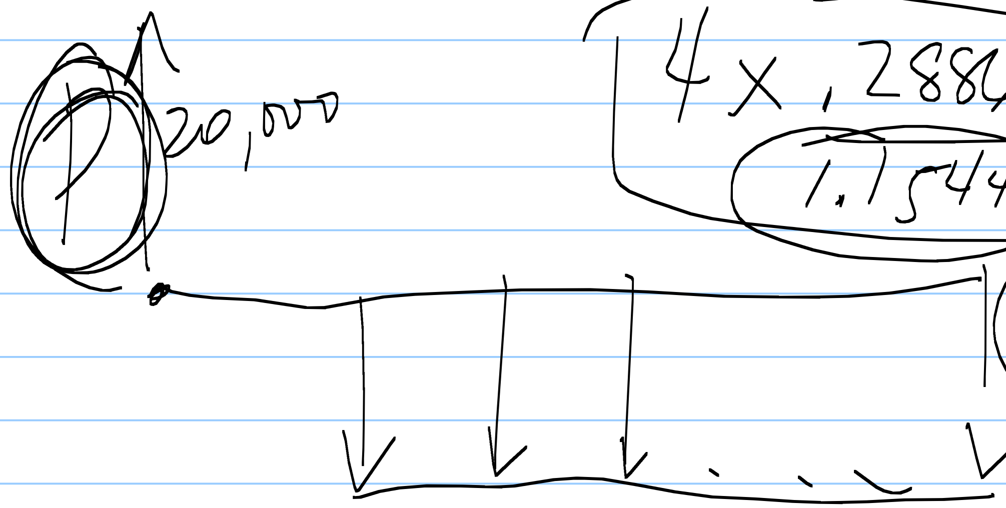
$i = 6\%$ annual compounded

$n = 48$ month

$4 \times .2886 \;(i = 6\%,\ n)^{4}$ ~~1.2625~~

1.1544

4x12



P 20,000

48

$A = ? \;(i = .5\%,\ 48)$

$48 \times .0235$

.0235
48
240
.44
96

$\boxed{1.1280}$

ENGINEERING ECONOMICS ISE460
SESSION 4
CHAPTER 3 continued, May 31, 2011

OUTLINE

- QUESTIONS?
- News?
  - See links page for credit card story
- CHAPTER 3 – continued
- Quiz results
- Questionnaire results
- Homework example
- Grades on line

EXAMPLE 3.25 Cash Flows with Subpatterns

The two cash flows in Figure 3.38 are equivalent at an interest rate of 12% compounded annually. Determine the unknown value C.

1100 . . . = . . 4C . C=275

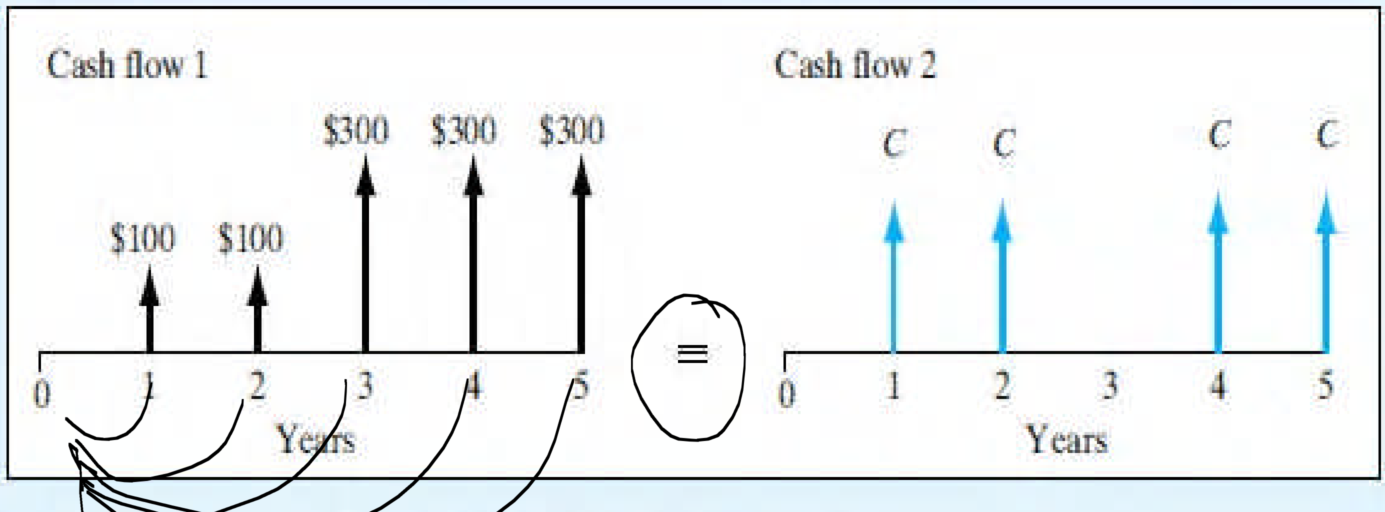
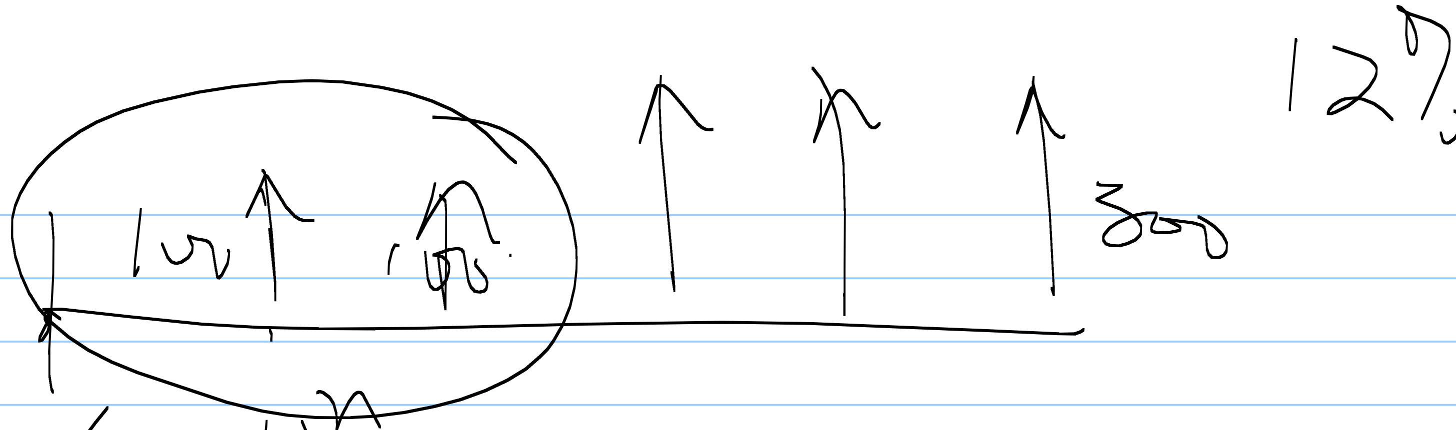


Figure 3.38 Equivalence calculation (Example 3.25).



$P = F(1+i)^{-n}$

$P_1 = 100(1+.12)^{-1}$ .8929

$P_2 = 100(1+.12)^{-2}$ .7972

$P_3 = 300(1+.12)^{-3}$ .7118 × 300   213.54

$P_4 = 300(1+.12)^{-4}$ .6355 × 300   190.65

$P_5 = 300(1+.12)^{-5}$ .5674 × 300   170.22

$\boxed{8.0332}$

8929

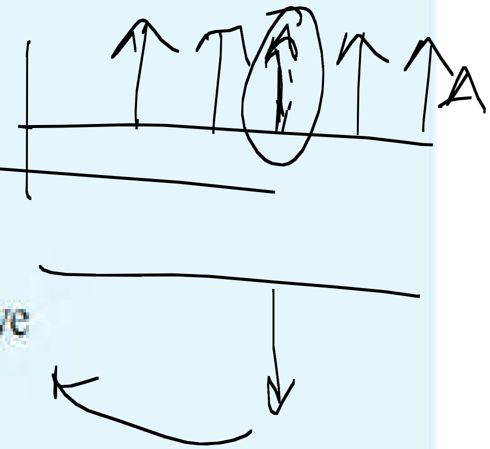
7972

3.25

- **Method 1.** Compute the present worth of each cash flow at time 0:

$$P_1 = \$100(P/A, 12\%, 2) + \$300(P/A, 12\%, 3)(P/F, 12\%, 2)$$
$$= \$743.42;$$

$$P_2 = C(P/A, 12\%, 5) - C(P/F, 12\%, 3)$$
$$= 2.8930C.$$

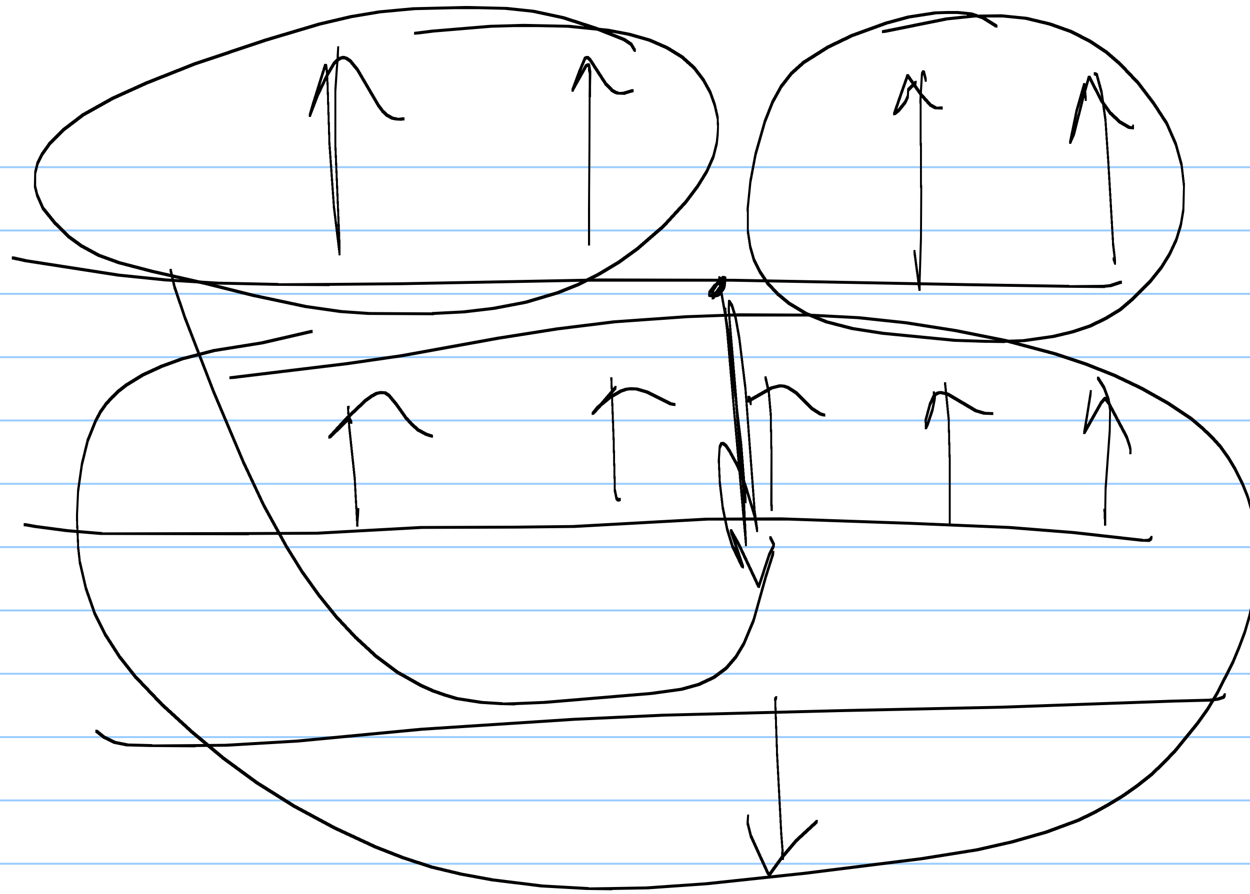


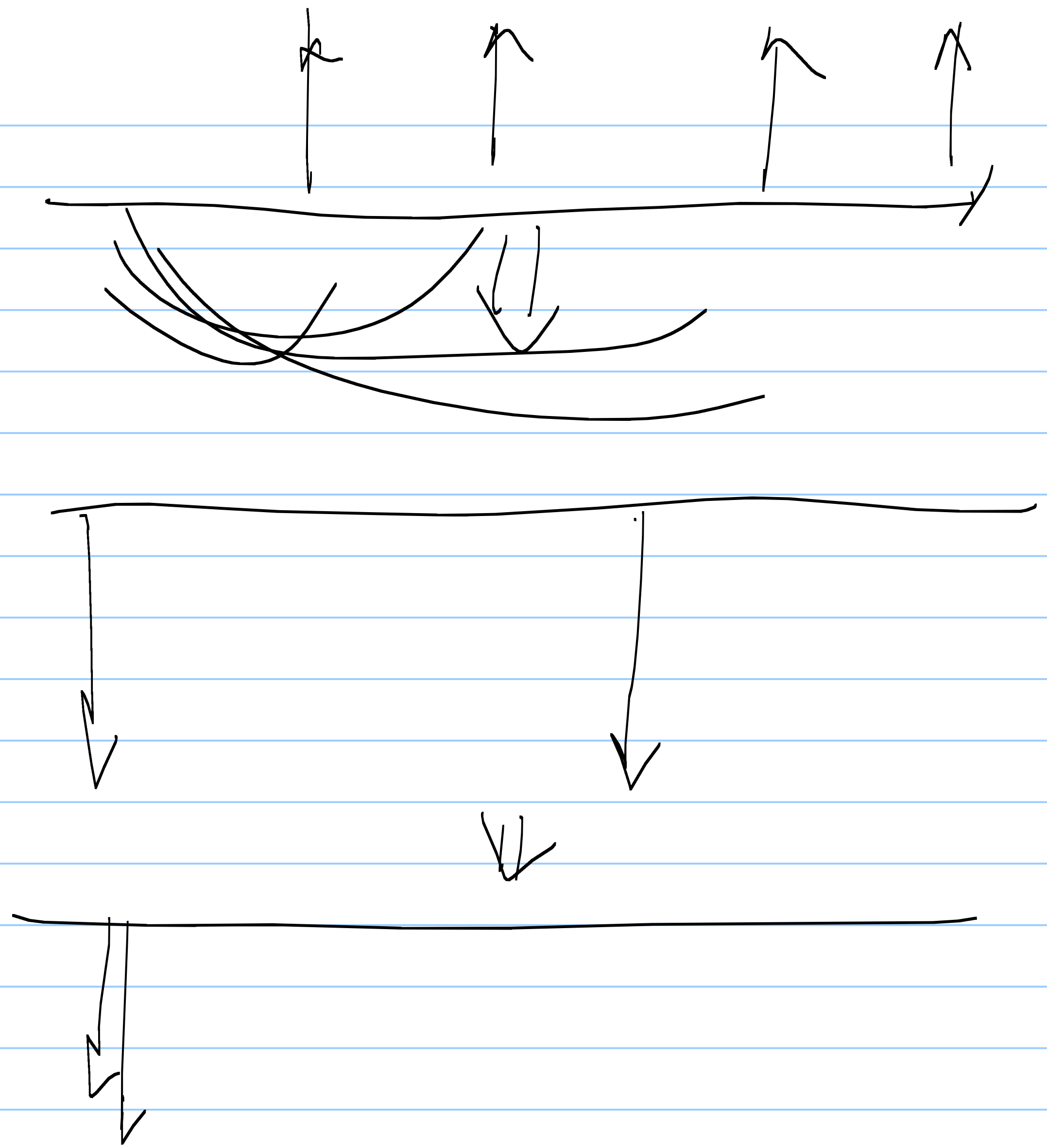
Since the two flows are equivalent, $P_1 = P_2$, and we have

$$743.42 = 2.8930C.$$

Solving for C, we obtain $C = \$256.97.$

275





P/F

3.26

EXAMPLE 3.26 Establishing a College Fund

33,999 A

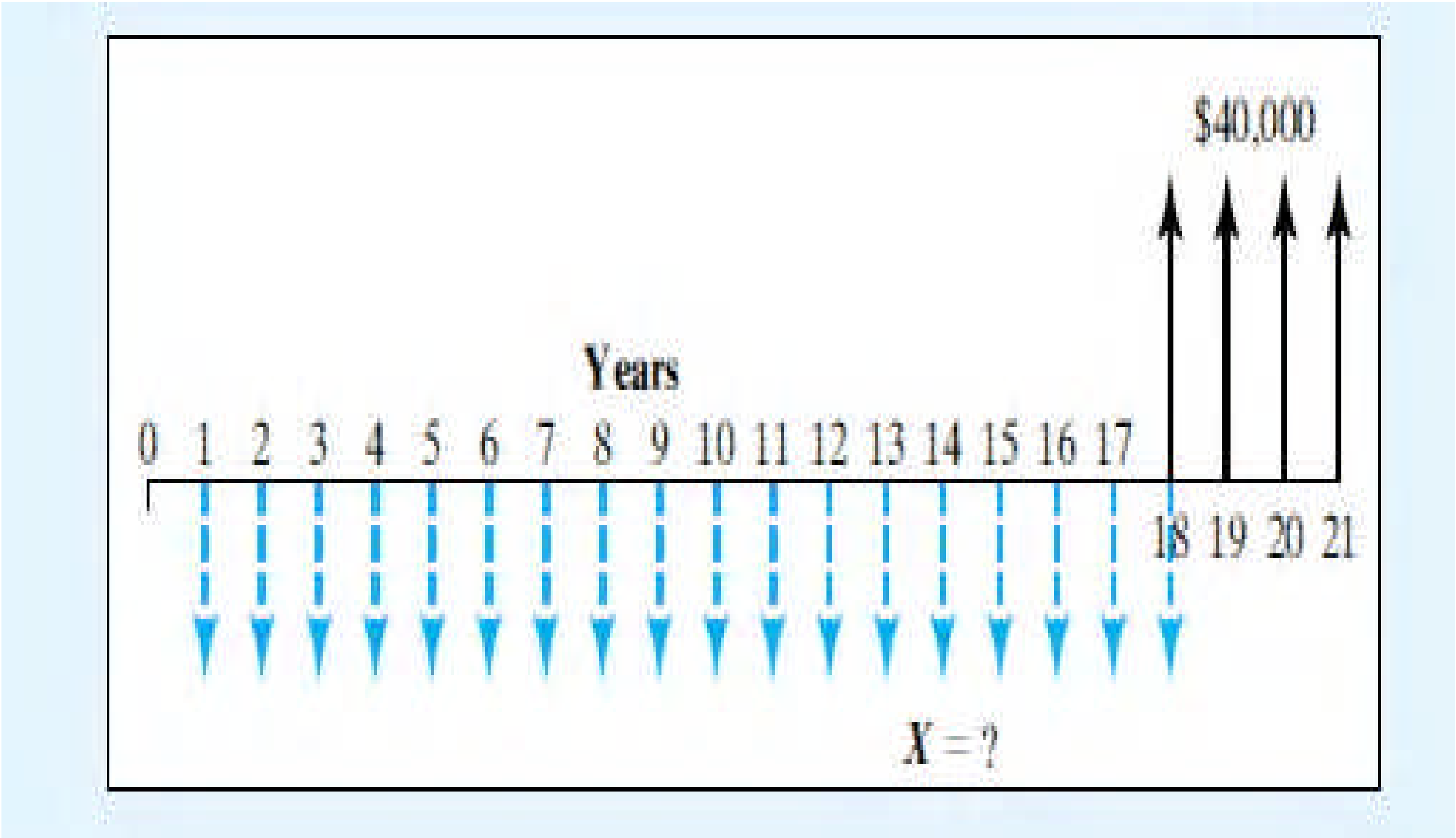
A couple with a newborn daughter wants to save for their child's college expenses in advance. The couple can establish a college fund that pays 7% annual interest. Assuming that the child enters college at age 18, the parents estimate that an amount of \$40,000 per year (actual dollars) will be required to support the child's college expenses for 4 years. Determine the equal annual amounts the couple must save until they send their child to college. (Assume that the first deposit will be made on the child's first birthday and the last deposit on the child's 18th birthday. The first withdrawal will be made at the beginning of the freshman year, which also is the child's 18th birthday.)

0 A 18 19 20 21

$F = A(i=7\%, n=18)$ 33,99

Geza P. Bottlik

3.26



3.26

- **Method 1.** Establish economic equivalence at period 0:

Step 1: Find the equivalent single lump-sum deposit now:

$$P_{\text{Deposit}} = X(P/A, 7\%, 18)$$
$$= 10.0591X.$$

Step 2: Find the equivalent single lump-sum withdrawal now:

$$P_{\text{Withdrawal}} = \$40{,}000(P/A, 7\%, 4)(P/F, 7\%, 17)$$
$$= \$42{,}892.$$

Step 3: Since the two amounts are equivalent, by equating $P_{\text{Deposit}} = P_{\text{Withdrawal}}$, we obtain X:

$$10.0591X = \$42{,}892$$
$$X = \$4{,}264.$$

Example 3.27

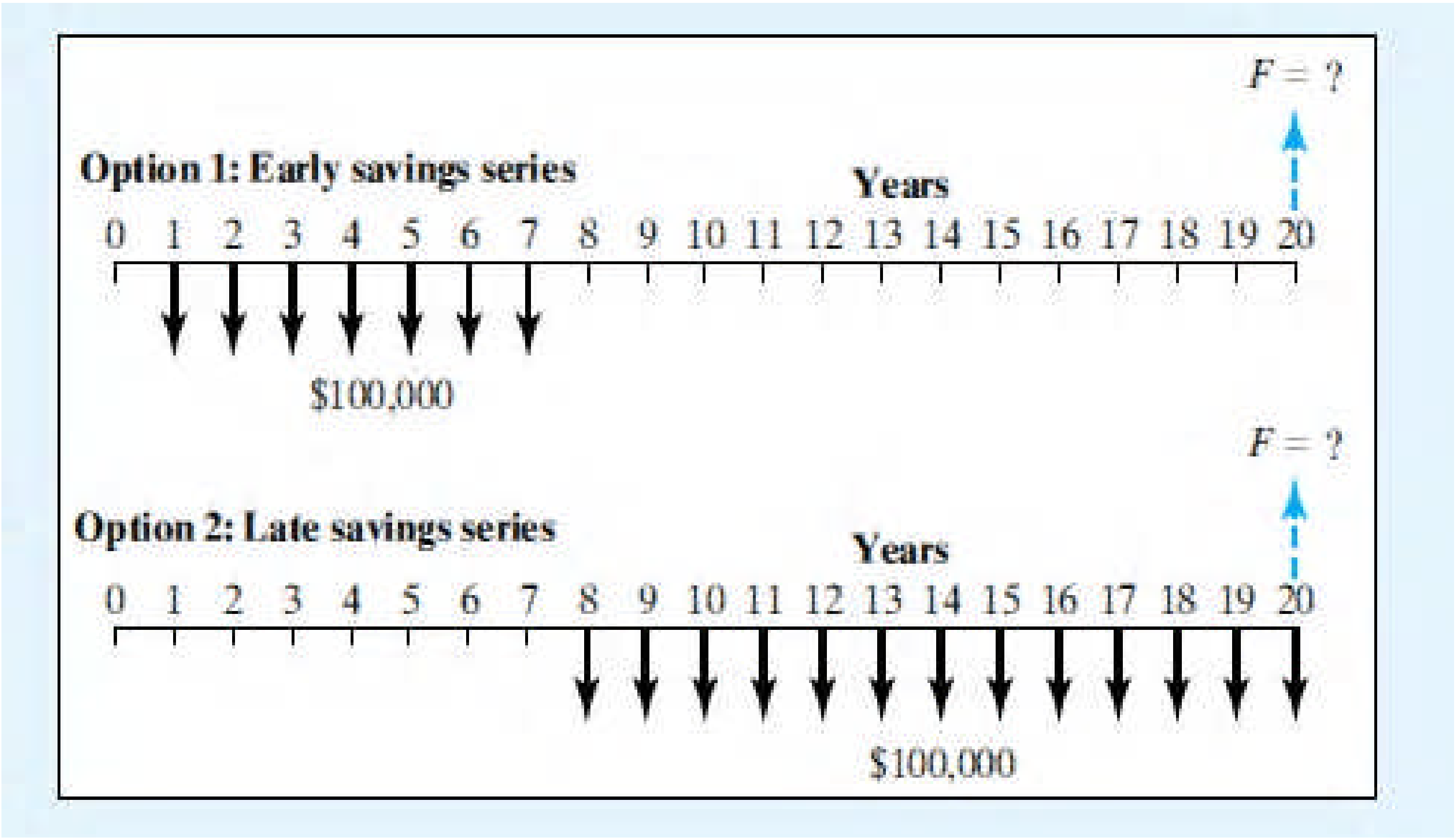
EXAMPLE 3.27 Calculating an Unknown Interest Rate with Multiple Factors

You may have already won $2 million! Just peel the game piece off the Instant Winner Sweepstakes ticket, and mail it to us along with your order for subscriptions to your two favorite magazines. As a grand prize winner, you may choose between a $1 million cash prize paid immediately or $100,000 per year for 20 years—that's $2 million! Suppose that, instead of receiving one lump sum of $1 million, you decide to accept the 20 annual installments of $100,000. If you are like most jackpot winners, you will be tempted to spend your winnings to improve your lifestyle during the first several years. Only after you get this type of spending "out of your system" will you save later sums for investment purposes. Suppose that you are considering the following two options:

Option 1: You save your winnings for the first 7 years and then spend every cent of the winnings in the remaining 13 years.
Option 2: You do the reverse, spending for 7 years and then saving for 13 years.

If you can save winnings at 7% interest, how much would you have at the end of 20 years, and what interest rate on your savings will make these two options equivalent? (Cash flows into savings for the two options are shown in Figure 3.41.)

3.27 – cash flows



3.27 – part a)

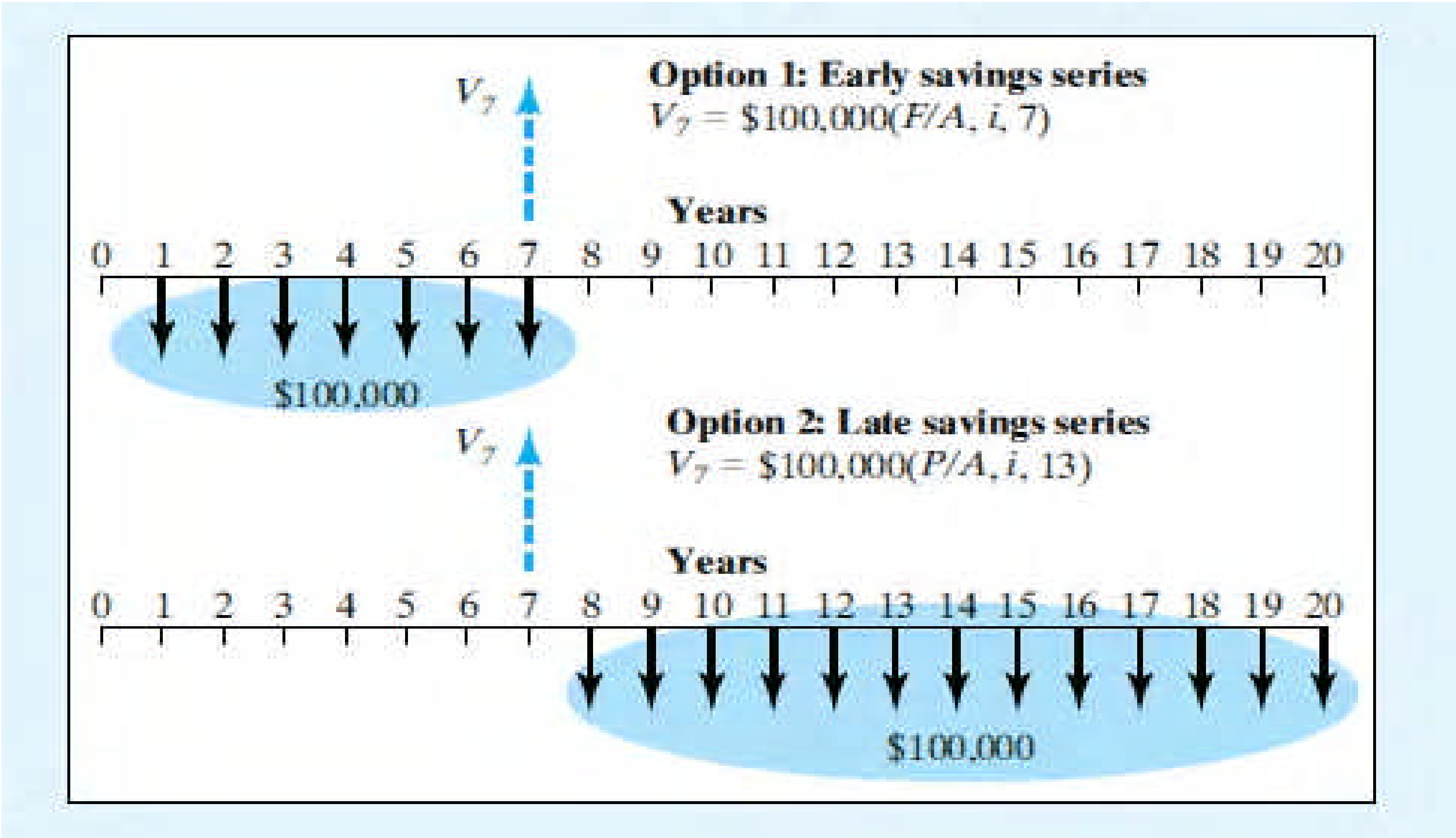
$$F_{\text{Option 1}} = \$100{,}000(F/A, 7\%, 7)(F/P, 7\%, 13)$$

$$= \$2{,}085{,}485;$$

$$F_{\text{Option 2}} = \$100{,}000(F/A, 7\%, 13)$$

$$= \$2{,}014{,}064.$$

3.27 part b)


Option 1: Early savings series
$V_7 = \$100{,}000(F/A, i, 7)$
V_7
Years
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
$100,000
Option 2: Late savings series
$V_7 = \$100{,}000(P/A, i, 13)$
V_7
Years
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
$100,000

3.27 part b)

- For Option 1,

$$V_7 = \$100{,}000(F/A, i, 7).$$

- For Option 2,

$$V_7 = \$100{,}000(P/A, i, 13).$$

We equate the two values:

$$\$100{,}000(F/A, i, 7) = \$100{,}000(P/A, i, 13);$$

$$\frac{(F/A, i, 7)}{(P/A, i, 13)} = 1.$$

3.27 part b)

Here, we are looking for an interest rate that gives a ratio of unity. When using the interest tables, we need to resort to a trial-and-error method. Suppose that we guess the interest rate to be 6%. Then

$$\frac{(F/A, 6\%, 7)}{(P/A, 6\%, 13)} = \frac{8.3938}{8.8527} = 0.9482.$$

This is less than unity. To increase the ratio, we need to use a value of i such that it increases the $(F/A, i, 7)$ factor value, but decreases the $(P/A, i, 13)$ value. This will happen if we use a larger interest rate. Let's try $i = 7\%$:

$$\frac{(F/A, 7\%, 7)}{(P/A, 7\%, 13)} = \frac{8.6540}{8.3577} = 1.0355.$$

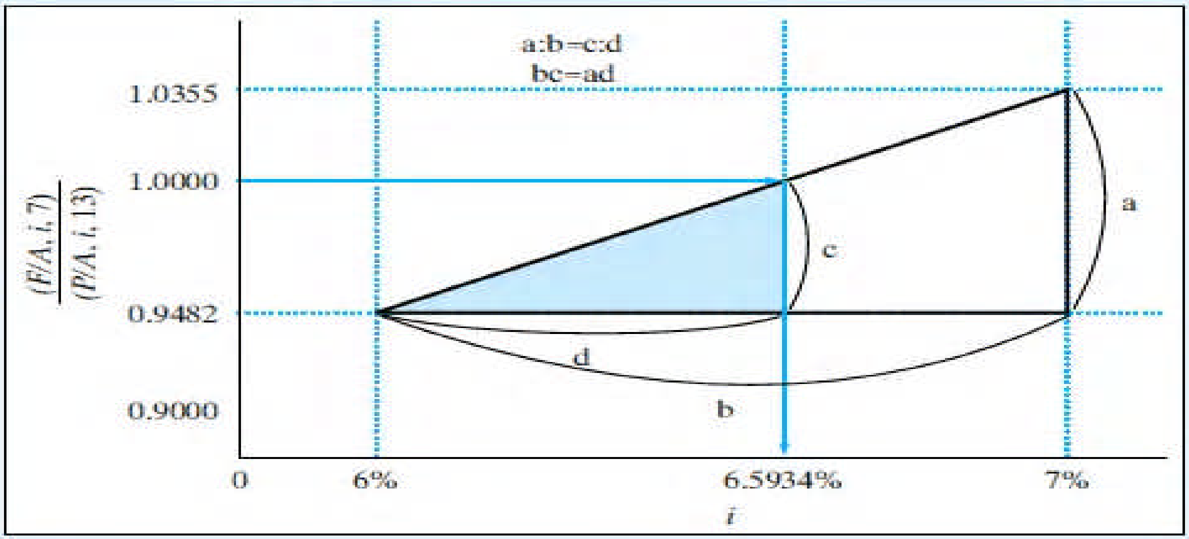
Now the ratio is greater than unity.

Interest Rate	$(F/A, i, 7)/(P/A, i, 13)$
6%	0.9482
?	1.0000
7%	1.0355

3.27 part b)

As a result, we find that the interest rate is between 6% and 7% and may be approximated by **linear interpolation** as shown in Figure 3.43:

$$i = 6\% + (7\% - 6\%)\left[\frac{1 - 0.9482}{1.0355 - 0.9482}\right]$$
$$= 6\% + 1\%\left[\frac{0.0518}{0.0873}\right]$$
$$= 6.5934\%.$$



Geza P. Bottlik