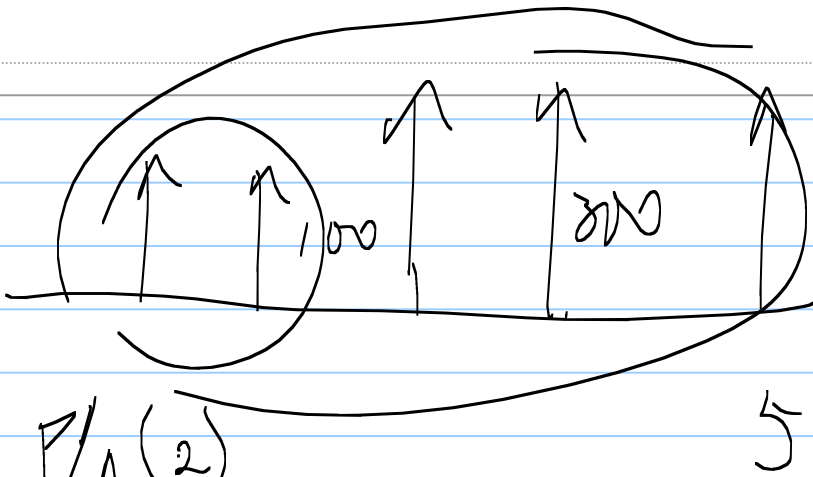
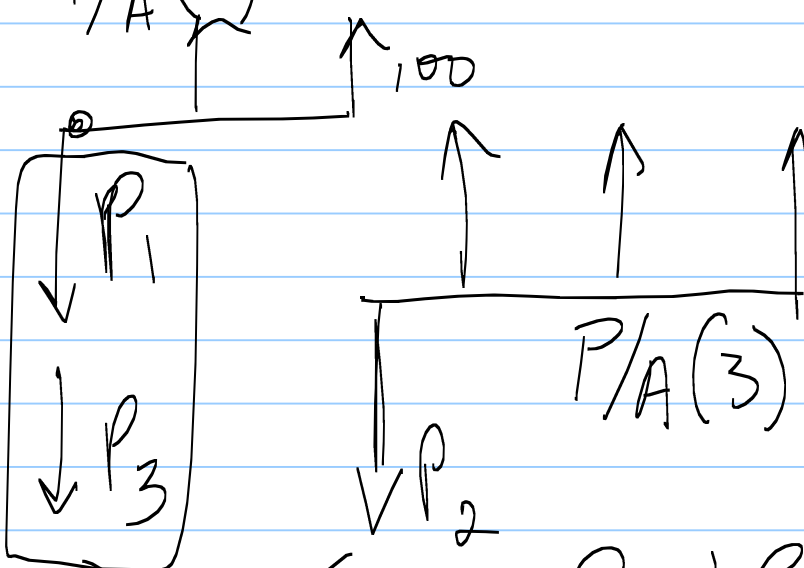
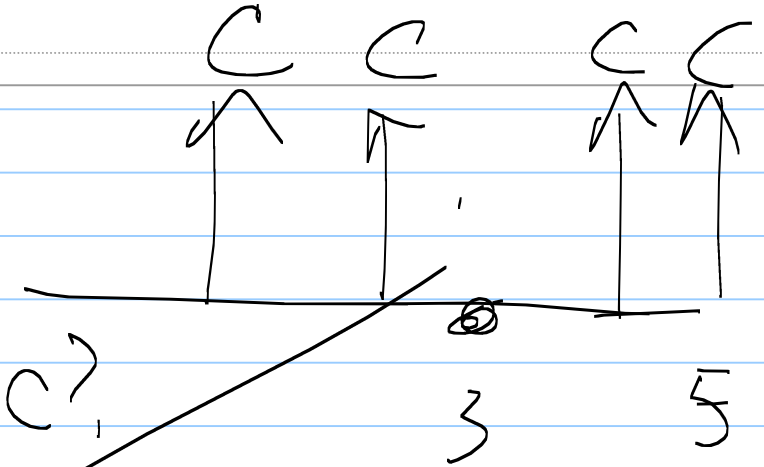


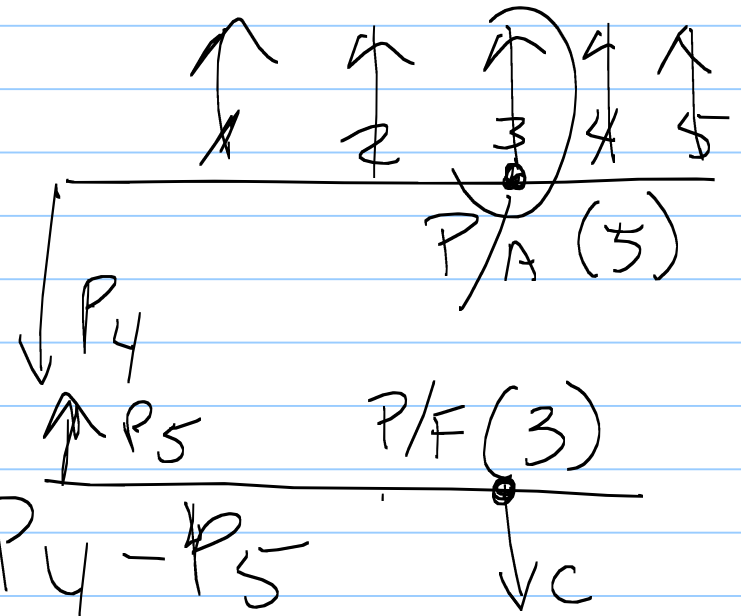
$i =$



$\delta$



300



$$P/F(2) \quad P_1 + P_3 = P_4 - P_5$$

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**OUTLINE**

- **Questions?**
- **Review Homework**
- **New Homework due 06/06**
- **Finish Quiz review**
- **Questionnaire results**
- **Chapter 4**
- **Excel demo of chapter 4 material**
  - **at least examples 4.1, 4.2, 4.12**
- **Quiz**

## WHAT'S IN CHAPTER 4?

**The difference between the nominal interest rate and the effective interest rate.**

- **The procedure for computing the effective interest rate, based on a payment period.**
- **How commercial loans and mortgages are structured in terms of interest and principal payments.**
- **The basics of investing in financial assets.**

## PROCEDURES

- **Effective Annual Interest Rates**
- **Equivalence Calculations with Effective Interest Rates**
- **Compounding Occurs at a Different Rate than That at Which Payments Are Made**
- **Changing Interest Rates**

# Debt Management

- **Commercial Loans**
- **Amortized Loans**
- **Add-On Interest Loans**
- **Loan versus Lease Financing**
- **Home Mortgage**

# Investing in Financial Assets

- **Basics**
- **Investing in Bonds**
- **Investing in Stocks**
- **Trading in Stocks**
- **Mutual Funds**
- **Commodities**

## CHAPTER 4 DEFINITIONS

- $r$  = NOMINAL INTEREST RATE PER YEAR = APR
- $M$  = COMPOUNDING FREQUENCY OR INTEREST PERIODS PER YEAR
- $r/M$  = INTEREST RATE PER COMPOUNDING PERIOD
- EFFECTIVE RATE = RATE EARNED IN A YEAR
- 18% APR = 1.5% PER MONTH =  $(1+0.015)^{12} = 1.196 = 19.6\%$
- $C$  = INTEREST PERIODS PER PAYMENT PERIOD
- $K$  = PAYMENT PERIODS PER YEAR ( $M = CK$ )

$$i_e = (1 + r / M)^M - 1$$

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**NOMINAL AND EFFECTIVE INTEREST RATES**

Nominal Rate	Compounding				
	Annually	Semi-Annually	Quarterly	Monthly	Daily
4%	4.00%	4.04%	4.06%	4.07%	4.08%
5%	5.00%	5.06%	5.09%	5.12%	5.13%
6%	6.00%	6.09%	6.14%	6.17%	6.18%
7%	7.00%	7.12%	7.19%	7.23%	7.25%
8%	8.00%	8.16%	8.24%	8.30%	8.33%
9%	9.00%	9.20%	9.31%	9.38%	9.42%
10%	10.00%	10.25%	10.38%	10.47%	10.52%
11%	11.00%	11.30%	11.46%	11.57%	11.63%
12%	12.00%	12.36%	12.55%	12.68%	12.75%
M	1	2	4	12	365

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**Example 4.1**

**EXAMPLE 4.1 Determining a Compounding Period**

The following table summarizes interest rates on certificates of deposit (CDs) offered by various lending institutions during November 2005:

Product	Bank	Minimum	Rate	APY*
3-Month CD	Imperial Capital Bank	\$2,000	4.03%	4.10%
6-Month Jumbo CD	IndyMac Bank	\$5,000	4.21%	4.30%
1-Year Jumbo CD	VirtualBank	\$10,000	4.50%	4.60%
1.5-Year CD	AmTrust Bank	\$1,000	4.50%	4.60%
2-Year CD	Ohio Savings Bank	\$1,000	4.59%	4.70%
2.5-Year Jumbo CD	Countrywide Bank	\$98,000	4.66%	4.77%
3-Year CD	ING Direct	\$0	4.70%	4.70%
5-Year CD	Citizens & Northern Bank	\$500	4.70%	4.78%

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**Example 4.1 solution**

**SOLUTION**

Given:  $r = 4.66\%$  per year,  $i_a$  (APY) =  $4.77\%$ ,  $P = \$100,000$ , and  $N = 2.5$  years.  
Find:  $M$  and the balance at the end of 2.5 years.

- (a) The nominal interest rate is  $4.66\%$  per year, and the effective annual interest rate (yield) is  $4.77\%$ . Using Eq. (4.1), we obtain the expression

$$0.0477 = \left(1 + \frac{0.0466}{M}\right)^M - 1,$$

or

$$1.0477 = \left(1 + \frac{0.0466}{M}\right)^M.$$

By trial and error, we find that  $M = 365$ , which indicates daily compounding. Thus, the 2.5-year Jumbo CD earns  $4.66\%$  interest compounded daily.

### Example 4.2

#### EXAMPLE 4.2 Effective Rate per Payment Period

Suppose that you make quarterly deposits in a savings account that earns 9% interest compounded monthly. Compute the effective interest rate per quarter.

#### SOLUTION

Given:  $r = 9\%$ ,  $C =$  three interest periods per quarter,  $K =$  four quarterly payments per year, and  $M = 12$  interest periods per year.

Find:  $i$ .

Using Eq. (4.2), we compute the effective interest rate per quarter as

$$\begin{aligned} i &= \left(1 + \frac{0.09}{12}\right)^3 - 1 \\ &= 2.27\%. \end{aligned}$$

### Example 4.3

#### EXAMPLE 4.3 Calculating an Effective Interest Rate with Quarterly Payment

Find the effective interest rate per *quarter* at a nominal rate of 8% compounded (a) quarterly, (b) monthly, (c) weekly, (d) daily, and (e) continuously.

#### SOLUTION

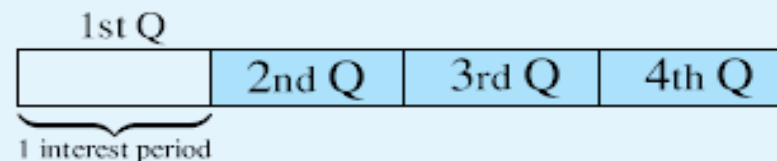
Given:  $r = 8\%$ ,  $M, C$ , and  $K = 4$  quarterly payments per year.

Find:  $i$ .

(a) Quarterly compounding:

$r = 8\%$ ,  $M = 4$ ,  $C = 1$  interest period per quarter, and  $K = 4$  payments per year. Then

$$i = \left(1 + \frac{0.08}{4}\right)^1 - 1 = 2.00\%.$$



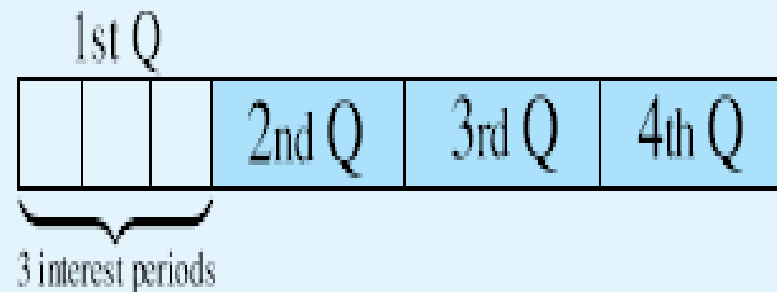
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Example 4.3 (cont)

(b) Monthly compounding:

$r = 8\%$ ,  $M = 12$ ,  $C = 3$  interest periods per quarter, and  $K = 4$  payments per year. Then

$$i = \left(1 + \frac{0.08}{12}\right)^3 - 1 = 2.013\%.$$



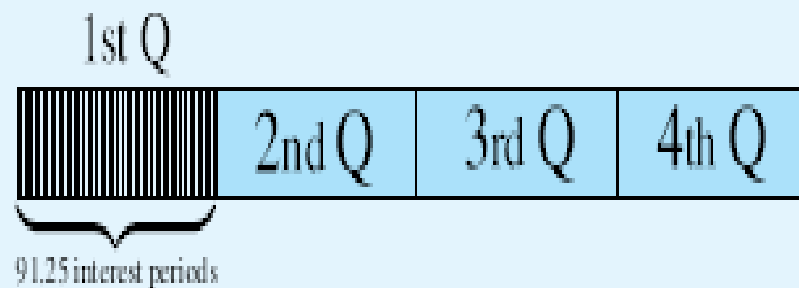
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Example 4.3 (cont)

(d) Daily compounding:

$r = 8\%$ ,  $M = 365$ ,  $C = 91.25$  days per quarter, and  $K = 4$ . Then

$$i = \left(1 + \frac{0.08}{365}\right)^{91.25} - 1 = 2.0199\%.$$



## AMORTIZED LOANS

- LOANS ARE PAID OFF IN INSTALLMENTS OR PAYMENTS:
- $B_n$  = REMAINING BALANCE AT THE END OF PERIOD  $n$
- $I_n$  = INTEREST PAYMENT IN PERIOD  $n$
- $P_n$  = PRINCIPAL PAYMENT AT THE END OF PERIOD  $n$

$$A_n = P_n + I_n$$

### EXAMPLE

\$5000 is borrowed at 12% APR for 2 years, to be paid monthly

$$A = P(A/P, 12\%/12, 24)$$

## ADD ON LOAN

- **TOTAL INTEREST (SIMPLE) TO BE PAID IS ADDED TO THE PRINCIPAL**
- **REPAID IN EQUAL INSTALLMENT**

### EXAMPLE 4.14

**Amount Borrowed = \$5000**

**Quoted simple annual interest rate = 12%**

**Length of loan = 2 years**

**Payment frequency = monthly**

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**Example 4.14 Solution**

(a) First we determine the amount of add-on interest:

$$iPN = (0.12)\$5,000(2) = \$1,200.$$

Then we add this simple-interest amount to the principal and divide the total amount by 24 months to obtain  $A$ :

$$A = (\$5,000 + \$1,200)/24 = \$258.33.$$

(b) Putting yourself in the lender's position, compute the APR value of the loan just described. Since you are making monthly payments with monthly compounding, you need to find the effective interest rate that makes the present \$5,000 sum equivalent to 24 future monthly payments of \$258.33. In this situation, we are solving for  $i$  in the equation:

$$\$258.33 = \$5,000(A/P, i, 24)$$

$$\text{then } (A/P, i, 24) = 0.0517$$

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**Example 4.14 Solution (continued)**

You know the value of the  $A/P$  factor, but you do not know the interest rate  $i$ .  
As a result, you need to look through several interest tables and determine  $i$  by interpolation.

A more effective approach is to use Excel's RATE function with the following parameters: =RATE (N,A,P,F,type,guess)

$$= \text{RATE}(24,258.33,-5000,0,0,1\%) = 1.7975\%$$

The nominal interest rate for this add-on loan is:

$$1.7975 * 12 = 21.57\%$$

and the effective annual interest rate is

$$1( + 0.01975)^{12} - 1 = \mathbf{26.45\%},$$

rather than the 12% quoted add-on interest. When you take out a loan, you should not confuse the add-on interest rate stated by the lender with the actual interest cost of the loan.

### Example 4.14 Solution (continued)

**COMMENTS:** In the real world, truth-in-lending laws require that APR information always be provided in mortgage and other loan situations, so you would not have to calculate nominal interest as a prospective borrower (although you might be interested in calculating the actual or effective interest). However, in later engineering economic analyses, you will discover that solving for implicit interest rates, or rates of return on investment, is performed regularly. Our purpose in this text is to periodically give you some practice with this type of problem, even though the scenario described does not exactly model the real-world information you would be given.

## **LEASING**

- **THREE WAYS TO BUY AN ASSET**
  - **CASH PURCHASE (YOU FOREGO INTEREST)**
  - **DEBT FINANCING (AMMORTIZED LOAN)**
  - **LEASING (RENT AND USE UP A PORTION OF THE VALUE)**
- **BUY VS. LEASE**
  - **EXAMPLE 4.17**
- **BUY OR FINANCE**
  - **EXAMPLE 4.16**

### Example 4.15

#### EXAMPLE 4.15 Financing your Vehicle: Paying Cash, Taking a Loan, or Leasing

Suppose you intend to own or lease a vehicle for 42 months. Consider the following three ways of financing the vehicle—say, a 2006 BMW 325 Ci 2-D coupe:

- **Option A:** Purchase the vehicle at the normal price of \$32,508, and pay for the vehicle over 42 months with equal monthly payments at 5.65% APR financing.
- **Option B:** Purchase the vehicle at a discount price of \$31,020 to be paid immediately.
- **Option C:** Lease the vehicle for 42 months.

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**Example 4.15**

Item	Option A Debt Financing	Option B Paying Lease Cash	Option C Financing	
Price		\$32,508	\$32,508	\$32,508
Down payment		\$4,500	\$0	\$0
APR(%)		5.65%		
Monthly payment		\$736.53		\$513.76
Length		42 months		42 months
Fees				\$994
Cash due at end of lease				\$395
Purchase option at end of lease				\$17,817
Cash due at signing		\$4,500	\$31,020	\$1,507.76

**Example 4.15**

**If the funds that would be used to purchase the vehicle are presently earning 4.5% annual interest compounded monthly, which financing option is a better choice?**

## **MORTGAGE -- DEFINITIONS**

- **SPECIAL LOAN FOR BUYING REAL ESTATE. THE REAL PROPERTY IS THE COLLATERAL. COLLATERAL MAY BE CLAIMED BY FORECLOSURE**
- **AMOUNT FINANCED**
- **TERM – RANGES FROM 15 YEARS TO 35 YEARS**
- **PAYMENT FREQUENCY – USUALLY MONTHLY**
- **INTEREST RATE**
- **POINTS – INTEREST CHARGED UP FRONT (SIMILAR TO ADD-ON)**
- **FEEES – COST ASSOCIATED WITH ESTABLISHING THE CONTRACT**

## **HOME MORTGAGE - EXAMPLES**

- **WITH OR WITHOUT POINTS**
- **MULTIPLE RATES**
- **VARIABLE RATE PEGGED TO ANOTHER RATE**
  - **ARIZONA HOME EXAMPLE**
- **REFINANCING - DISCUSSION**

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**Example**

- **Borrow \$100,000 for 15 years, making monthly payments**
- **Option 1: Interest rate of 6.375% APR, pay 1 point (%) that is subtracted up front (you only get \$99,000, but have to pay back \$100,000)**
- **Option 2: Interest rate of 6.75% APR**

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**VARIABLE RATES**

Dear Borrower:

RE: Loan No.

The terms of your adjustable mortgage loan provide for the following changes:

The payment of your mortgage loan is scheduled to be adjusted for the payment due on 11/01/02. This new payment consists of a principal and interest amount of \$768.47. This payment amount is based on a rate of 5.50000%, a remaining term of 200 months and a projected principal balance of \$100,484.96. This does not include payment for taxes and insurance(s).

Assuming that all regularly scheduled payments (if any) are made between now and 11/01/02, your balance will be \$100,484.96.

The interest rate effective 10/01/02 used to calculate the new payment was 5.50000%.

This interest rate was based on the FHLB OF SAN FRANCISCO 11TH DISTRICT COST OF FUNDS index value of 2.84700% published on 07/31/02.

Your next scheduled payment review date is 02/15/03.

Any new payment amount will be due for the first time on 05/01/03.

The interest rate on your mortgage loan is scheduled for adjustment prior to your next payment review. The dates your interest rate will be adjusted during this period begin on 11/01/02 and every 6 month(s) thereafter.

Since your last payment adjustment, the interest rate on your mortgage loan has been adjusted on the following dates:

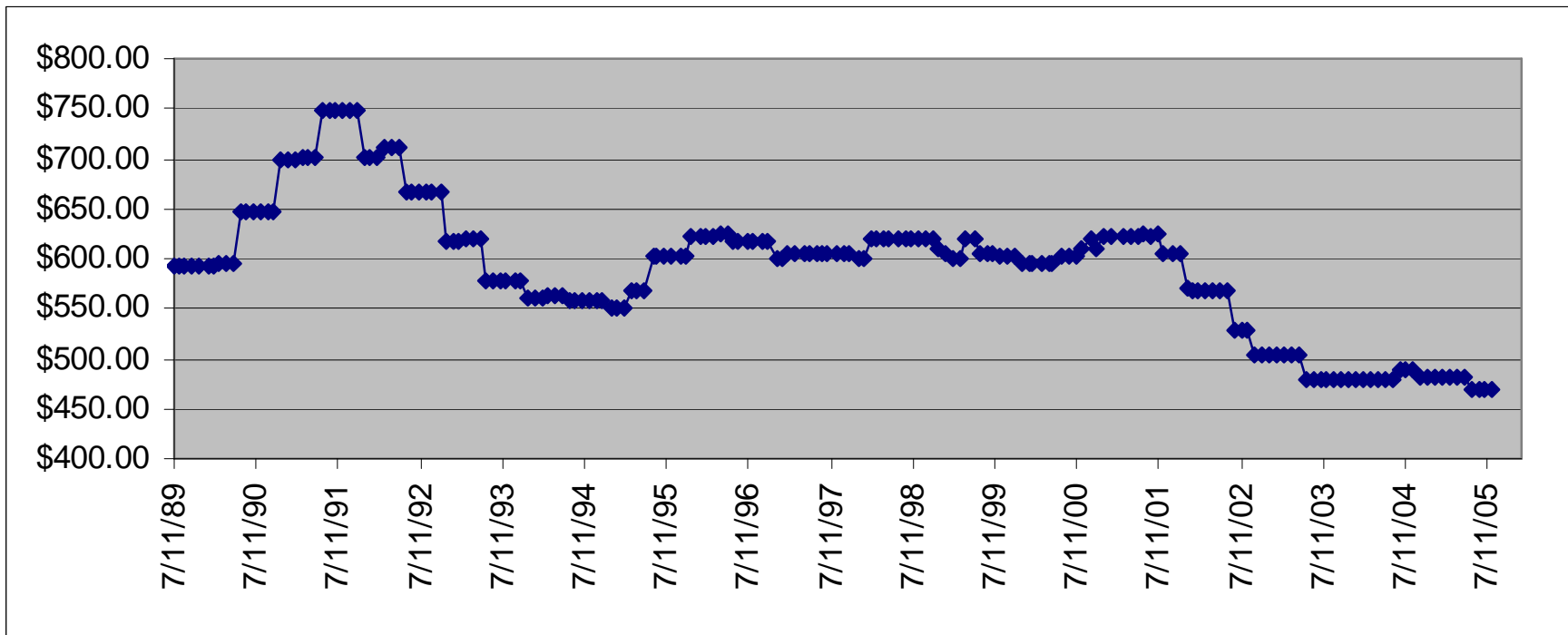
PREVIOUS ADJUSTMENT DATE	PREVIOUS INTEREST RATE	PREVIOUS INDEX VALUE USED
05/01/02	6.25000%	3.07400%

For prompt service when writing or calling about this account, please use this address: P.O. Box 1093, Northridge, CA 91328-1093; or phone a Customer Service Representative at (866) 469-2651. Always provide your loan number 5935006865.

Washington Mutual

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VARIABLE RATES



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**Example**

- **Borrowing \$100,000, with 5% down payment**
- **Given 4 different interest rates**
  - **The first three last 1 year each**
  - **The fourth is for the balance of 27 years**
- **The assumption is that the borrowers ability to pay will increase during the first few years**

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**Example**

	A	B	C	D	E	F	G	H	I	J	K	L	M
52	<b>5.19</b>											30	
53		8.125%	12	0.6771%			\$95,000	360			(\$705.37)	<b>PMT(D53,H53,G53)</b>	
54		10.125%	12	0.8438%			\$94,225.88	348			(\$840.17)		
55		12.125%	12	1.0104%			\$93,658.39	336			(\$979.77)		
56		13.125%	12	1.0938%			\$93,234.22	324			(\$1,050.71)		
57													
58				1.0059%	12.76%		\$7,936.08				(\$982.32)		
59							\$8,382.91						
60							\$8,669.49						
61							\$70,011.52				<b>PV(D58,36,0,-PV(D58,H56,K56))</b>		
62							\$95,000.00						
63													
64													
65													
66													
67													
68													

**Goal Seek** ? X

Set cell:

To value:

By changing cell:

## **INVESTING**

- **LIQUIDITY – HOW QUICKLY YOU CAN GET AT YOUR MONEY**
  - **FROM IN YOUR POCKET TO TIED UP IN A 35 YEAR MORTGAGE (ALTHOUGH YOU COULD TRY AND SELL THE MORTGAGE)**
- **RISK – HOW LIKELY ARE YOU TO LOSE THE VALUE OF YOUR ASSET**
  - **FROM U.S. TREASURY BONDS TO BETTING ON HORSES**
- **RETURN – PROFIT OVER TIME**
  - **SUM OF THREE FACTORS**
    1. **“RISK FREE” REAL RETURN**
    2. **INFLATION FACTOR**
    3. **PREMIUM FOR RISK**

## INVESTING

- RETURN (CONTINUED)
  - COMPOUND
  - AVERAGE ANNUAL RATE (SEE EXAMPLE 6.1)
- VOLATILITY – HOW MUCH DOES IT MOVE UP OR DOWN
  - +/- \$ OR %
  - STANDARD DEVIATION =  $s$  (\$ OR  $z$ )
    - » MEASURES PROBABILITY FROM THE MEAN
      - +/- 1  $s$  = 65%
      - +/- 2  $s$  = 95%
      - +/- 3  $s$  = 99%

## INVESTING

- **STRATEGY – RISK VS. PAYOFF**
  - **DOLLAR AVERAGING – INVEST AT A STEADY RATE REGARDLESS OF PRICE**
  - **DIVESIFICATION – SPREAD THE RISK AMONG MANY DIFFERENT INVESTMENTS**
  - **ASSET ALLOCATION- MOVE YOUR MONEY AMONG DIFFERENT INVESTMENTS – THIS IS WHAT MONEY MANAGERS DO**

## DEFINITIONS

- **MUTUAL FUND – POOLED MONEY IN MANY STOCKS**
- **STOCK – PART OWNERSHIP IN A COMPANY. YOU PAY A COMMISSION TO A BROKER TO BUY OR SELL A STOCK. THE PRICE DEPENDS ON WHAT OTHERS ARE WILLING TO PAY FOR IT**
- **DIVIDEND -- A PORTION OF THE PROFITS PAID TO THE OWNERS OF SHARES**
- **CAPITAL GAIN (LOSS) – WHAT YOU MAKE OR LOSE ON THE SALE OF AN ASSET HELD OVER 6 MONTHS**
- **SEE EXAMPLE 6.3 FOR AN ATTEMPT TO EVALUATE A STOCK (NOT ON THE SPREADSHEET DEMO)**

## **BONDS**

- **ISSUED BY:**
  - **COMPANIES**
  - **FEDERAL GOVERNMENT (SAVINGS BONDS)**
  - **STATE GOVERNMENTS**
  - **MUNICIPAL GOVERNMENTS (USUALLY TAX FREE)**

**BONDS (CONTINUED)**

- **DEFINITIONS**
  - **MORTGAGE BONDS = BACKED BY SPECIFIC COLLATERAL**
  - **DEBENTURES = BACKED BY OVERALL ASSETS**
- **PAR VALUE = STATED FACE VALUE**
- **MATURITY DATE = DATE ON WHICH PAR VALUE WILL BE REPAYED**
- **COUPON RATE - INTEREST PAID ON THE PAR VALUE**
  - **SEMIANNUAL PAYMENT IS USUAL**

**BONDS (CONTINUED)**

- **DEFINITIONS (CONTINUED)**
- **DISCOUNT BOND - SOLD BELOW ITS PAR VALUE**
- **PREMIUM BOND - SOLD ABOVE ITS PAR VALUE**
- **MARKET PRICE - THE PRICE AT WHICH THE BOND IS CURRENTLY BOUGHT AND SOLD**

**DEFINITIONS(CONTINUED)**

- **OPTION – AN OPPORTUNITY TO BUY A STOCK**
  - **BUY PUT – TO BUY WITHIN A SPECIFIED TIME**
  - **BUY CALL – TO BUY AT A GIVEN PRICE (STRIKE PRICE) WITHIN A SPECIFIC TIME**
  - **EXERCISE AN OPTION – BUY THE STOCK**